Understanding the Efficiency of Autonomous Nano- and Microscale Motors

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Supporting Information

ABSTRACT: We analyze the power conversion efficiency of different classes of autonomous nano- and micromotors. For bimetallic catalytic motors that operate by a self-electrophoretic mechanism, there are four stages of energy loss, and together they result in a power conversion efficiency on the order of 10^{-9}. The results of finite element modeling agree well with experimental measurements of the efficiency of catalytic Pt--Au nanorod motors. Modifications of the composition and shape of bimetallic catalytic motors were predicted computationally and found experimentally to lead to higher efficiency. The efficiencies of bubble-propelled catalytic micromotors, magnetically driven flagellar motors, Janus micromotors driven by self-generated thermal gradients, and ultrasonically driven metallic micromotors are also analyzed and discussed.

INTRODUCTION

Nano- and microscale motors are tiny objects that are capable of converting the energy of chemical fuels or external fields into mechanical motion.1,2 Such motors are ubiquitous in the biological world, ranging from enzymatic assemblies that are nanometers in size to bacteria and cells in the range of micrometers to tens of micrometers. Synthetic motors that are similar in size to bacteria were first introduced in 2004–2005.3,4 The first generation of these motors were 2–3 μm long bimetallic rods (Pt−Au and Ni−Au) that catalytically decomposed hydrogen peroxide to oxygen and water and moved at speeds in the range of 10 μm/s. Over the past decade, bimetallic catalytic motors have been redesigned for faster movement5–9 and for functionality that includes sensing, pumping, and cargo delivery.10–13 Microjets are a related class of tubular or conical catalytic motors that also catalyze hydrogen peroxide decomposition.14–21 Unlike bimetallic nanorods, which are propelled by self-electrophoretic forces, microjets are propelled by bubble recoil and travel at impressively fast speeds, up to mm/s.22 They have recently been studied for potential applications that include cell sorting and transport.20,23–25 Magnetically driven motors are another emerging class of micromachines26–31 which, like flagellar bacteria, convert the whiplike or rotary motion of their tails into axial translation.

While much effort has been devoted to increasing the speed and functionality of autonomous motors, few papers have commented on the efficiency with which they convert the free energy stored in fuel to mechanical energy. A power conversion efficiency on the order of 10^{-9} was estimated by Paxton et al. for Au−Pt nanorod catalytic motors in hydrogen peroxide solutions.23 In contrast to these low experimentally measured efficiencies, the movement of nanodimers and catalytic nanoparticles has been studied in molecular dynamics simulations, and a maximum thermodynamic efficiency on the order of 10^{-4} was found for both systems.33,34 However, the small size of these hypothetical nanomotors and their operation under idealized conditions (separate dimers with attractive or repulsive interdimer potentials and catalytic particles in gaseous fuels, respectively) makes it difficult to compare them with experimentally studied nano/micromotors. Sabass and Seifert found computationally that the efficiency of diffusiophoretic nano/micromotors should increase with decreasing motor size.35,36 However, there are so far no studies that attempt to model and also measure the efficiencies of synthetic nano/micromotors (self-electrophoretic, bubble-propelled, magnetic, self-thermophoretic, and ultrasonically driven) that have been experimentally fabricated.

In contrast to synthetic motors, the efficiency of nano- and micromotors in biology is typically high. For example, kinesin-based microtubule motors have been estimated to have a chemical-to-mechanical power conversion efficiency of roughly 60%.37,38 The efficiency of biological motors confers many advantages, including the ability to run for long periods of time using on-board fuel, and to use oxidants that are present in low concentration or consumed in slow catalytic reactions. For example, aerobic organisms are able to use oxygen in metabolic reactions despite its low concentration (0.3 mM) in air-saturated water. Because synthetic nano/micromotors are many...
orders of magnitude less efficient, they are usually studied in solutions that contain high concentrations (~1 M) of hydrogen peroxide or other fuels. A broader choice of fuels (including, for example, glucose/oxygen) would be available to catalytic motors and pumps if their efficiency could be increased by 3–4 orders of magnitude or more. Understanding the sources of energy loss in these systems is thus important to broadening their range of useful properties and applications.

**MATERIALS AND METHODS**

**Synthesis of Nanorods and Nanotubes.** Bimetallic nanorods were fabricated as described in earlier publications.\(^3\) Anodized alumina membranes (AAO, purchased from Whatman Inc., 200 nm nominal pore size) were used as templates for the electrodeposition of metals. The metal plating solutions were purchased from Technic, Inc. A 5 nm thick film of Cr and a 350 nm thick film of Ag were evaporated onto the branched pore side (hereafter called the back side) of the membranes by using a Kurt Lesker Lab-18 evaporator. This metal film served as the working electrode contact for electrodeposition. The lengths of the metal segments were controlled by monitoring the charge passed. The plating conditions for Ag, Au, and Pt were \(-1.77, -1.24,\) and \(-1.77 \text{ mAh/cm}^2,\) respectively. Ru was deposited at a constant potential of \(-0.65 \text{ V vs Ag/AgCl. Multisegment nanorods were synthesized by replacing the plating solution without disassembling the plating cell, with a water rinsing step in between. After the electrodeposition step, the membrane was thoroughly rinsed with deionized (DI) water and air-dried, and typically one-half of the membrane was soaked sequentially in 1:1 v/v HNO\(_3\) and 0.5 M NaOH etch and was then heated at 80°C overnight. Together with the etching of the side wall, this heating step created a small gap between the polyaniline nanorods and the pore wall, thus enabling the etching of the side wall, this heating step created a small gap between the polyaniline nanorods and the pore wall, thus enabling the deposition of metal nanotubes. The metals of interest were then sequentially deposited into the gap following ordinary electrodeposition procedures. Polyaniiline was removed with concentrated HNO\(_3\) after the deposition of the metal tubes.

**Motor Tracking.** The detailed procedure for tracking motor movement is described in earlier papers.\(^3\) Bimetallic nanorods were suspended in aqueous 5% H\(_2\)O\(_2\) solution for at least 15 min before they were taken for observation. An Olympus BX60M optical microscope (reflection mode) and a commercial video capturing bundle (Dazzle video creator plus) were used for observing the particles and recording videos, and typically an overall magnification of 500× or 1000× was used to take each video clip of 30 s at 30 frames/s. Dark field is ideal for observing the motion of nanorods due to the reflective nature of the metals. In most cases, the observation cell was a piece of rectangular glass capillary tubing (VitroTubeTM from VitroCom, 0.2 × 2.00 mm, catalog # 3520-050), which was filled with the nanorod suspension by capillarity.

The videos were then loaded into PhysMo 2, an open source tracking program (PhysMo - Video Motion Analysis Package, http://physmo.sfn.net), and the coordinates of the metal rods were recorded as a function of time. Rod speed was calculated by dividing the displacement of the rod center between sequential frames by the time interval (0.033 s), and then taking the average of the speed over the selected tracking period. The tracking was repeated with multiple rods (at least 10 data points for each rod) to ensure statistically robust results.

In the experiments in which the wall effect was examined more closely, a different observation cell was used. In this case, a ring-shaped piece of double sided sticky tape was attached to a piece of glass slide (poly(ethyleneglycol) (PEG) coated or uncoated). A volume of 50 µL of nanorod suspension in 5% H\(_2\)O\(_2\) solution was added to the center of the ring, and a glass coverslip was placed on top to seal the cell. The uncoated glass slide was precleaned in Piranha solution (concentrated H\(_2\)SO\(_4\)/30% H\(_2\)O\(_2\) = 3:1. **Caution! Extremely corrosive! Protective equipment and care are needed when handling Piranha solutions!**). Au–Pt nanomotors moved at an average speed of 9 ± 2 and 14 ± 3 µm/s in cells made from uncoated glass slides and PEG-coated glass slides, respectively.

**Oxygen Evolution Measurements.** Nanorods of known concentration were mixed with aqueous 5% H\(_2\)O\(_2\) for 15 min before each experiment. The suspension was transferred to a 10 mL test tube, which was sealed and purged with Ar before sampling. For each sample, 500 µL Ar was injected into the tube and a 500 µL gas sample was withdrawn and injected into a gas chromatograph. The oxygen evolution rate per wire was calculated after taking into account the oxygen from the air that had leaked in, the volume of gas in the head space and the number of wires in the suspension. A control experiment was carried out in the absence of catalytic nanorods with all the other experiential parameters the same. A typical result is shown in Figure S-1 (see Supporting Information).

**RESULTS AND DISCUSSION**

**Self-Electrophoretic Catalytic Motors.** While a number of mechanisms have been proposed to contribute to the propulsion of bimetallic motors in H\(_2\)O\(_2\) solutions, there is good evidence that self-electrophoresis (Figure 1) is dominant for motors in the size range of a few micrometers and below.\(^1,\)\(^3\) In this mechanism, the oxidation and reduction of H\(_2\)O\(_2\) occur preferentially at the anode (Pt) and cathode (Au), respectively (eqs 1 and 2). This results in an asymmetric distribution of protons near the ends of the rod, which in turn creates an electric field in the solution near the metal rod. Because the metal surface is negatively charged, the rod moves closely, a dipeptide was used to take each video clip of 30 s at 30 frames/s.

![Figure 1. Illustration of the self-electrophoretic propulsion mechanism for a Au–Pt catalytic nanomotor. Red and blue represent high and low proton concentrations, respectively. The electric field E points from the anode (Pt) end to the cathode (Au). The white arrow indicates the direction of motion of the motor.](https://example.com/figure1.png)

Because the metal surface is negatively charged, the rod moves closely, a dipeptide was used to take each video clip of 30 s at 30 frames/s.

- **anode:** \(H_2O_2(aq) \rightarrow 2H^+(aq) + O_2(g) + 2e^-\) (1)
- **cathode:** \(H_2O_2(aq) + 2H^+(aq) + 2e^- \rightarrow 2H_2O(l)\) (2)
- **cathode:** \(\frac{1}{2}O_2(g) + 2H^+(aq) + 2e^- \rightarrow H_2O(l)\) (3)
overall: \(2\text{H}_2\text{O}_2(\text{aq}) \rightarrow 2\text{H}_2\text{O}(l) + \text{O}_2(\text{g})\) \hspace{1cm} (4)

The power conversion efficiency of the nanomotor is defined by eq 5,

\[
\eta_{\text{overall}} = \frac{\text{mechanical power output}}{\text{total chemical power input}} = \frac{P_{\text{mecha}}}{P_{\text{chem}}}
\]

where \(P_{\text{mecha}}\) and \(P_{\text{chem}}\) are further defined as

\[
P_{\text{mecha}} = F_{\text{drag}}\nu = f0^2
\]

For spheres

\[
f = 6\pi\mu r
\]

For cylinders

\[
f = \frac{2\pi\mu L}{\ln\left(\frac{L}{r}\right) - 0.72}
\]

\[
P_{\text{chem}} = n_{\text{O}_2}\Delta_r^G
\]

Here \(F_{\text{drag}}\) is the drag force on a spherical or cylindrical motor, \(\mu\) is the dynamic viscosity of water, \(r\) is the radius of the spherical particle, \(L\) is the length of the cylindrical rod (3 \(\mu\)m), \(R\) is its radius (150 nm), \(\nu\) is the motor speed, \(n_{\text{O}_2}\) is the oxygen evolution rate in units of mol/(rod s), and \(\Delta_r^G\) is the Gibb's free energy of the decomposition of \(\text{H}_2\text{O}_2\) into water and oxygen (eq 4, \(\sim 206\text{kJ/mol}\) of \(\text{O}_2\) produced). The mechanical work (\(= P_{\text{mecha}} \times \text{time}\)) is the useful work done by the motors. From the \(\text{O}_2\) evolution rate of Au–Pt nanorods in 5% \(\text{H}_2\text{O}_2\) (Figure 2, left column), we obtained \(n_{\text{O}_2} = 2.3 \times 10^{-15}\text{mol/(rod s)}\) and \(\Delta_r^G = 80\text{kJ/mol}\).

For cylinders

\[
\frac{10^{-15}\text{mol/(rod s)}}{2.3 \times 10^{-15}} = 4.6 \times 10^{-15}\text{mol/(rod s)}
\]

The oxygen evolution rate (Figure 2) and average axial velocity, the efficiency could be calculated from eqs 6b and 7. The speed of the Au–Pt nanomotors was \(21 \pm 4\text{ \mu m/s}\), yielding an overall efficiency (sometimes referred to as a thermodynamic efficiency) of \(7 \times 10^{-5}\), in rough agreement with previous report of Paxton et al. The fastest moving self-electrophoretic nanomotors reported so far have a speed of \(\sim 150\mu\text{m/s}\), with an Ag–Au alloy as the cathode. If these motors consume fuel at the same rate as Au–Pt nanorods, their energy efficiency would be on the order of \(10^{-7}\).

There are four sources of loss that together result in the very low efficiency of bimetallic catalytic nanomotors. The first stage of energy loss comes from the chemical (or nonelectrochemical) decomposition of \(\text{H}_2\text{O}_2\), which is catalyzed by Pt. The fuel utilization efficiency can be expressed as

\[
\eta_{\text{fuel}} = \frac{\text{electrochemical decomposition rate}}{\text{overall decomposition rate of fuel}}
\]

The electrochemical decomposition rate was measured by Paxton et al. using Pt–Au interdigitated microelectrodes in \(\text{H}_2\text{O}_2\) solutions of various concentrations, and a current density of \(0.68\text{A/m}^2\) was found for 6% \(\text{H}_2\text{O}_2\). Although the geometry of that experiment was different from that of Au–Pt nanorods, the microelectrode current density provides a rough estimate of the electron transfer rate (and proton flux) at the nanomotor surface. Using this value, we obtain \(7 \times 10^{-5}\text{mol/(m}^2\text{s)}\) for these fluxes, and the electrochemical \(\text{H}_2\text{O}_2\) decomposition rate can be calculated as \(0.5\sim1.1 \times 10^{-7}\text{mol/(rod s)}\), depending on the balance of reactions 2 and 3 at the cathode. From the total decomposition rate of \(\text{H}_2\text{O}_2\) (4.6 \(\times 10^{-15}\text{mol/(rod s)})\), we then obtain \(\eta_{\text{fuel}} = 1.1 \times 10^{-9}\) to \(2.3 \times 10^{-9}\). Because Wang et al. found that eq 3 is the dominant cathode reaction, we estimate \(\eta_{\text{fuel}}\) to be approximately \(1 \times 10^{-3}\) for Au–Pt nanorod motors.

It is apparent from this calculation that the efficiency of nanorod motors could be increased by a factor of \(\sim 10^3\) by using metals that selectively catalyze \(\text{H}_2\text{O}_2\) oxidation and reduction without catalyzing the overall decomposition reaction. Substitution of Ru for Pt lowers the rate of \(\text{O}_2\) evolution by 80% (Figure 2) and Au–Ru nanorods move at similar speed (18 \(\pm 3\mu\text{m/s}\)) to Au–Pt (21 \(\pm 4\mu\text{m/s}\)), resulting in a 4-fold increase in efficiency. However, Ru is still very catalytically active and decomposes roughly 99% of the fuel chemically, rather than electrochemically. A recent report by Liu and Sen describes self-electrophoretic Cu–Pt nanomotors that operate in dilute \(\text{Br}_2\) or \(\text{I}_2\) solutions. Since the spontaneous reaction of \(\text{Br}_2\) or \(\text{I}_2\) at the Cu surface of the Cu–Pt nanomotors is not significant, these nanomotors are \(\sim 10^3\) times more efficient than Pt–Ru nanorods in \(\text{H}_2\text{O}_2\) solutions and are thus able to move at speeds up to 20 \(\mu\text{m/s}\) in solutions of \(\sim 2\text{mM halogen}\).

A second stage of energy loss arises from the use of a very exerogenic reaction to generate a small potential drop along the surface of the nanomotor. The electrochemical potential of the cell (eq 4) can be calculated as 1.07 V from eq 9:

\[
\Delta_r^G = -nFE_{\text{cell}}
\]

In contrast, the potential difference (\(\Delta\phi\)) in the solution between the two ends of a Au–Pt nanomotor, at a surface proton flux of \(7 \times 10^{-3}\text{mol/(m}^2\text{s)}\), was found by finite element simulations (see the Supporting Information) to be only 2.1 mV. A cell efficiency term is defined by eq 10 to represent this stage of energy loss:

\[
\eta_{\text{cell}} = \frac{\Delta\phi}{E_{\text{cell}}} = \frac{2.1\text{mV}}{1.07\text{V}}
\]

For the Au–Pt catalytic nanomotors, \(\eta_{\text{cell}}\) is \(\sim 2 \times 10^{-3}\). It is interesting to contrast this value with the near-unit efficiency of biological motors that are driven by proton gradients, such as ATP synthetase. The biological motors operate near the...
reversible limit through a tight coupling of mechanical work to the movement of protons down (or up) a pH gradient. In the present case, the proton gradient and resulting electric field established by the reaction are depolarized by convection, by electromigration, and especially by rapid diffusion of protons from the anode to the cathode, as we show below.

Referring to Figure 1, oxidation and reduction occur preferentially at the Pt and Au ends of the nanorods, respectively. Protons are generated at the Pt end and consumed at the Au end, leading to a concentration gradient of protons around the rod (see Figure S5a for the numerically simulated proton concentration profile). The concentration gradient drives protons by diffusion from the anode to the cathode. In addition, protons are subject to migration in the same direction in the self-generated electric field, and also move by convection in the frame of the moving rod. The counterion HCO$_3$ is subject to the same effects (the solution is exposed to the air and CO$_2$ is dissolved in the solution; see eq S4 in the Supporting Information), but its migration is in the opposite direction. The transport of ions is governed by the general flux equation (eq 11),

$$J_i = u_i c_i - D_i \nabla c_i - \frac{z_i D_i F \epsilon_i \nabla \phi}{RT}$$  

(11)

where $J_i$ is the flux of ion $i$ and the three terms on the right represent convection, diffusion, and electromigration, respectively. Far from the surface of the motor, the flux and each of the gradient terms are zero, but at the motor surface $J_{\text{flux}}$ is related to the current density $j$ by $J_{\text{flux}} = j / F$. Here $u$ is the fluid velocity, $\phi$ is the electrostatic potential, $F$ is the Faraday constant, $R$ is the gas constant, $T$ is the absolute temperature, and $c_i$, $D_i$, and $z_i$ are the concentration, diffusion coefficient and charge of species $i$, respectively.

The respective contributions of diffusion, electromigration and convection to the transport of protons can be estimated by numerical simulation (Figure 3; see the Supporting Information for details). For protons moving along the long axis of the nanorod, electromigration is dominant, contributing ~68% of the overall flux. The diffusive flux is ~29%, and convection is relatively unimportant at ~3% (Figure 3b). On the other hand, for proton transport away from and toward the nanorod (perpendicular to the direction of motion), diffusion accounts for nearly 99% of the overall flux (Figure 3c), while proton transport by electromigration and convection are negligible.

This simulation shows that the diffusion and electromigration fluxes, which are both proportional to $D_{i,\text{flux}}$, the diffusion coefficient of protons (eq 11), are together responsible for the dissipation of the proton gradient generated by the electrochemical reaction at the Pt–Au nanorod surface. Thus, the value of $D_{i,\text{flux}}$, which is 9.3 x $10^{-5}$ cm$^2$/s in pure water at 25 °C, can affect the speed and efficiency of the motor. By varying the diffusion coefficient of protons in the numerical model, the calculated motor speed could be varied by a factor of 3, as shown in Figure 4. Moran et al. found that the speed of self-electrochemically driven motors is inversely proportional to the proton diffusion coefficient in their scaling analysis of the Au–Pt nanomotor system. Although the result obtained by numerical modeling in Figure 4 is not strictly inversely proportional, it shows a similar trend. We note that the purpose of this simulation is to demonstrate how diffusion coefficient affects the motor behavior; experimentally, the diffusion coefficient of any ion in water cannot surpass that of protons.

Experimentally, it is difficult to manipulate the proton diffusion coefficient without significantly altering other parameters of the system, such as viscosity or temperature. However, it is possible to limit proton diffusion by spatial confinement. In order to test this hypothesis, we fabricated bimetallic nanotubes in which both the inner and outer surfaces

![Figure 3](attachment:figure3.png)

**Figure 3.** Numerical simulation to determine the contributions of diffusion, electromigration, and convection to proton transport. (a) Steady-state proton distribution around a Pt (top)-Au (bottom) catalytic nanomotor (3 μm long and 300 nm in diameter) with a surface flux of 7 x $10^{-6}$ μmol/(m$^2$.s). Red and blue represent high and low proton concentrations (units in legend: mol/m$^3$). Arrows illustrate proton flux contributions (arrows lengths are not in exact proportion to the flux). The units for the $x$ axis are μm. (b) Proton fluxes from diffusion, electromigration, and convection along the nanorod long axis across the center line (dashed line in (a)). (c) Proton fluxes perpendicular to the nanorod surface from the top half (Pt end, across the black line in (a)). In (b) and (c), fluxes were integrated over the respective cross sections.
were catalytically active. These tubular nanomotors, as a result, were predicted to generate a higher proton flux and also inhibit diffusion of protons out of the tube, thus significantly increasing the electric field and consequently the motor speed.

Numerical modeling of these tubular nanomotors shows that they indeed trap protons inside the tube and increase the local proton concentration significantly (see Figures S11 and S12). We fabricated Au–Ru nanotubes by a modification of the template-assisted electrodeposition process described by Shin et al.43 Figure 5 shows field emission SEM images and a cartoon of the structure of these nanotubes.

![Figure 4](image1.png)

Figure 4. Relationship between the proton diffusion coefficient (\(D_{H^+}\)) and the motor speed from numerical simulations. \(D_{H^+}/D_{H^+0}\) is the experimental proton diffusion coefficient in water. In the model, the \(D_{H^+}/D_{H^+0}\) ratio was arbitrarily varied between 0.1 to 10.

![Figure 5](image2.png)

Figure 5. FESEM images of Au–Ru nanotubes: (a) side and (b) end-on view.

The Au–Ru nanotubes moved at a speed of 32 ± 5 μm/s in 5% \(H_2O_2\) solution, roughly 75% faster than solid Au–Ru nanorods of similar dimensions (18 ± 3 μm/s). The nanotubes moved with their Au end leading, and no bubbles were observed, consistent with the self-electrophoretic mechanism. Gas chromatographic measurements showed that the rates of \(O_2\) evolution from nanotubes and nanorods were the same, consistent with the self-electrophoretic mechanism. We fabricated Au–Ru nanotubes by a modification of the template-assisted electrodeposition process described by Shin et al.43 Figure 5 shows field emission SEM images and a cartoon of the structure of these nanotubes.

Figure 6. Equivalent circuit of the nanomotor-solution system. The blue cylinder around the bimetallic nanorod represents the solution near its surface, which is represented as a resistor in the equivalent circuit on the right.

![Figure 6](image3.png)

For this stage we can define the propulsion efficiency by eq 12:

\[
\eta_{prop} = \frac{P_{mecha}}{P_{elec}} = \frac{P_{mecha}}{\Delta \phi I}
\]

where \(P_{mecha}\) is the mechanical power of the nanomotor (3.3 \(\times\) \(10^{-18}\) W/rod from eq 6), \(P_{elec}\) is the electrical power that is generated by the calculated 2.1 mV potential difference (\(\Delta \phi\)), and \(I\) is the current flowing in the system. Taking the current density to be 0.68 A/m\(^2\) and the surface area of each segment of the rod as 1.48 \(\times\) \(10^{-12}\) m\(^2\), the current is 1.0 pA. Therefore, \(P_{elec}\) is 2.1 \(\times\) \(10^{-15}\) W/rod and \(\eta_{prop}\) is 1.6 \(\times\) \(10^{-3}\), or on the order of 10\(^{-3}\). This component of the overall motor efficiency is sometimes referred to as the hydrodynamic efficiency.

The inherently low efficiency of this stage can be understood in terms of the electrophoretic force acting on a charged particle. This is most simply modeled for a Janus sphere of radius \(r\), for which the electrophoretic velocity \(v\) is

\[
v \sim \frac{\zeta e_i E}{\mu} \sim \frac{\zeta e_i e_0 \Delta \phi}{\pi \eta \mu}
\]

Here, \(\zeta\) is the zeta potential of the particles, which Paxton et al. measured as ~40 mV for Au rods in deionized water.11 The electric field \(E\) can be approximated as the potential drop (\(\Delta \phi\)) divided by the path length \(l\) around the sphere, \(\pi r\). The other terms in the equation are \(e_i\), the relative permittivity of water (80.1), \(e_0\) the vacuum permittivity (8.854 \(\times\) \(10^{-12}\) F/m), and \(\mu\), the viscosity of water (1 \(\times\) \(10^{-3}\) N·s/m at 293 K). The drag force and the mechanical power of the Janus sphere are given by eq 6.

The electrical power can be calculated as the current–voltage product, that is,

\[
P_{elec} = jA \Delta \phi = 2\pi r^2 \Delta \phi
\]
where \( j \) is the current density and \( A \) is the area of one-half of the Janus sphere, which equals \( 2\pi r^2 \). Combining eqs 13 and 14, we obtain the efficiency as

\[
\eta_{\text{prop}} = \frac{P_{\text{mech}}}{P_{\text{dec}}} = \frac{6\pi\mu r^2}{2\pi r^2 j} \approx \frac{3(\zeta_\epsilon \epsilon_0)^2 \Delta \phi}{\pi^2 r^2 \mu j} \quad (15)
\]

The last term in this equation can be simplified by expressing \( \Delta \phi/j \) in terms of the resistivity \( \rho \) of the solution, which Paxton et al. measured as \( 2.5 \times 10^3 \Omega \cdot \text{m} \) for suspensions of gold nanorods in water.11 Making the simplifying assumption that ions have the same current densities in the system, we have

\[
\frac{\Delta \phi}{j} = \frac{A \Delta \phi}{I} = AR \approx A \left( \frac{\rho l}{A'} \right) \approx \pi \rho
\]

Combining these equations, we obtain

\[
\eta_{\text{prop}} \approx \frac{3(\zeta_\epsilon \epsilon_0)^2 \rho}{\pi^2 \mu} \quad (16)
\]

Substituting the values given above with \( r = 1 \mu m \), we calculate \( \eta_{\text{prop}} \approx 2 \times 10^{-3} \), in reasonable agreement with the measured value. This result shows that electrophoretic energy transfer in the micrometer (and larger) size regimes, especially in electrolyte solutions with low \( \rho \) (e.g., high ionic strength). The 1/\( r^2 \) scaling of \( \eta_{\text{prop}} \) suggests that submicrometer swimmers might have higher efficiency. However, \( A/A' \) will also decrease as \( r \) decreases, to some extent canceling the effect.

The fourth stage of energy loss for bimetallic catalytic motors is the electroosmotic flow of fluid at the bottom of the glass cell, which opposes the direction of electrophoretic propulsion for negatively charged nanorods. Because the nanorods are made of dense metals, they sink to an equilibrium position slightly above the bottom wall of the cell. Their height above this glass surface is determined by a balance of the downward gravitational force and the electrostatic repulsion between the rod and the glass. The electroosmotic counter-flow is illustrated in Figure 7.

![Figure 7](image)

Figure 7. A self-electrophoretic nanomotor generates an electric field, which induces electroosmotic flow along the charged wall of the cell in the opposite direction.

Because of the close proximity of the nanomotor to the wall (500 nm to 1 \( \mu m \)), the effect of electroosmotic flow can be significant. In fact, there are cases where local electroosmotic flow dominates over the electrophoretic transport of particles and causes them to move in the opposite direction.14\(,48,49 \) To further confirm the existence of the reverse electroosmotic flow and to estimate its effect on the motors, we compared the movement of Au–Pt nanomotors suspended in 5% \( H_2O_2 \) over an uncoated glass slide and one coated with poly(ethylene glycol) (PEG). The PEG coating imparts a smaller (~10 mV) charge to the surface.50\(–52 \) Nanomotors moved ~60% faster over this surface than they did over uncoated glass slides, which typically have zeta potentials of ~60 to ~90 mV.53 Taking into account that the mechanical power of the nanomotor scales as the square of its velocity (eq 6), we can define an efficiency related to this wall effect:

\[
\eta_{\text{wall}} = \left( \frac{v}{v_0} \right)^2 \quad (18)
\]

where \( v \) is the observed velocity of nanomotors with the wall effect and \( v_0 \) is the velocity of motors in its absence of the effect. Based on the difference in the speeds for nanomotors moving near walls of different zeta potentials, we can estimate \( \eta_{\text{wall}} \leq 0.4 \).

This effect can be minimized by lowering the charge density of the wall, as illustrated in the PEG experiment. However, the nanorods are maintained above the wall by electrostatic propulsion, and they adhere to walls of opposite charge. Thus, a more thorough analysis is needed to evaluate the optimal surface charge on the bottom wall that gives the highest efficiency. An alternative is to increase the negative surface charge on the nanomotor, which should increase its speed and, through electrostatic repulsion, its distance from the bottom wall.

The energy loss due to the wall effect is approximately a factor of 2, which is less significant than the other three energy loss pathways, each of which have values of about \( 10^{-3} \). The wall effect can however be important for large and/or dense micromotors. In such cases, the distance between the motor and the wall is small compared to the size of the motor. In the presence of a large cargo particle, depending on its dimensions and density, the wall effect could generate stronger backward electroosmotic flow or hydrodynamic drag that significantly slows down the motor.

Catalytic Bubble Motors. It is interesting to compare the efficiency of self-electrophoretic motors with those that utilize other propulsion mechanisms. In two cases, the propulsion efficiencies are relatively straightforward to estimate: micromotors driven by bubble propulsion18,17,18,54 and magnetically driven helical micromotors.6,28,29 These two types of motors have been well studied and are being investigated for a variety of applications, including those in biological environments.20,22,24,25,55

Tubular catalytic micromotors, first reported by Solovev et al.,14 are propelled by oxygen bubbles generated by the catalytic decomposition of \( H_2O_2 \). Motors of different dimensions and with different bubble production rates result in different velocities. For example, multilayered 5.5 \( \mu m \) diameter microtubes 100 \( \mu m \) in length were reported to move through 3% \( H_2O_2 \) solution at 50 \( \mu m/s \). In 3% \( H_2O_2 \), the microtubes generate 34 \( \mu m \) radius bubbles at a rate of 1.5 per second. Using the ideal gas law, the oxygen evolution rate is \( 1.10 \times 10^{-11} \) mol/(tube-s), corresponding to a chemical input power of \( 2.27 \times 10^{-6} \) W/tube. The mechanical power output can be calculated from the velocity (eq 6c), to be \( 5.46 \times 10^{-16} \) W/tube. The ratio gives an efficiency of \( 2.40 \times 10^{-10} \), which is comparable to that of self-electrophoretic nanomotors. A similar analysis applied to bubble-powered Janus sphere micromotors56 gives a calculated efficiency of \( 5 \times 10^{-10} \).

The low efficiency of bubble-powered micromotors can be understood in terms of two stages of energy loss. The expansion of the oxygen gas into bubbles large enough to detach from the particle surface is a prerequisite to motion. The
expansion power of the oxygen bubbles can be estimated, assuming the ideal gas law and reversible expansion, to be 2.50 × 10^{-8} W/jet. Thus, the work of bubble expansion constitutes ~1% of the chemical input energy, and 99% of the energy of the reaction is dissipated as heat at this stage. The second stage of energy loss derives from the fact that, at low Reynolds number, propulsion happens only at the instant of bubble release. Careful examination of microtube motion has shown that the motors are stationary except for sharp spikes of movement corresponding to bubble release.\(^\text{19}\) At low Reynolds number (10^{-4}−10^{-5}), the time scale of acceleration and deceleration is on the order of microseconds, and the recoil movement persists only over this short time period. Further, because the momentum of the bubble and motor must be equal and opposite at the instant of release, some of the kinetic energy goes into the bubble and the viscous layer of water between its surface and the shear plane. The combination of these effects apparently results in a second-stage efficiency of ~10^{-8} for bubble motors, making their overall efficiency ~10^{-10}.

**Magnetic Micromotors.** It is also interesting to consider the efficiency of magnetic micromotors, which incorporate flexible chains made of superparamagnetic colloidal particles,\(^\text{31}\) partially dissolved nanowires,\(^\text{30}\) or helix shaped microparticles.\(^\text{26,28}\) This class of micromotors is especially interesting because of their resemblance to flagella bacteria, which combine the helical form of the synthetic motors with catalytic propulsion. In an externally applied rotating magnetic field, helical magnetic motors rotate. The torque applied to the helical tail translates into an axial force, which results in directional motion of the micromotors. The torque, axial force, linear velocity, and angular velocity of the motor can be related via a 2 × 2 Purcell matrix\(^\text{57}\)

\[
\begin{bmatrix}
F \\
N
\end{bmatrix} = \begin{bmatrix}
a & b \\
b & c
\end{bmatrix} \begin{bmatrix}
\nu \\
\Omega
\end{bmatrix}
\]

(19)

where \(F\) is the axial force (which equals the drag force on the micromotor at low Reynolds number), \(N\) is the torque (in N·m) due to the applied magnetic field, \(\nu\) is the axial speed of the motor, and \(\Omega\) is the angular speed (in rad/s). This matrix gives the efficiency, \(\eta\):

\[
\eta = \frac{\text{mechanical power output}}{\text{power input}} = \frac{F\nu}{N\Omega}
\]

(20)

This calculation does not take into account the magnetic energy absorbed by the medium surrounding the motor, or the heat generated in the electrical circuit that controls the magnet. It accounts only for the mechanical energy imparted by magnetic induction and the fraction of that energy that is converted to axial movement. Thus, \(\eta\) represents an upper limit for the efficiency of the motor.

Among several reports on helical magnetic micromotors, one by Nelson et al. provides details about the Purcell matrix, axial force, and torque.\(^\text{26}\) For a 38 \(\mu\)m micromotor in a 2 mT magnetic field, the Purcell matrix was found to be

\[
\begin{bmatrix}
a & b \\
b & c
\end{bmatrix} = \begin{bmatrix}
1.5 \times 10^{-7} & -1.6 \times 10^{-14} \\
-1.6 \times 10^{-14} & 2.3 \times 10^{-19}
\end{bmatrix}
\]

(21)

As a result the maximum drag force, torque, angular velocity, and motor velocity were derived as 3.0 \times 10^{-12} N, 4.3 \times 10^{-17} N·m, 190 rad/s, and 1.8 \times 10^{-7} m/s, respectively. These values give \(\eta = 0.66\%\), that is, on the order of \(10^{-2}−10^{-3}\). This is slightly higher than the hydrodynamic propulsion efficiency of electrophoretically (10^{-3}) or diffusiophoretically driven (10^{-3})\(^\text{15,36}\) motors in the micrometer size range.

The hydrodynamic efficiency of these helical motors is comparable to that of flagellar bacteria. Purcell calculated that the maximum efficiency of rotating flagella is ~1%.\(^\text{58}\) The experimental efficiency of *E. coli* propulsion has been estimated to be around 2%.\(^\text{59,60}\) Purcell has pointed out that the energy consumption of a flagellar bacterium is negligible compared to its energy intake in an environment in which nutrition is abundant. Propulsion mechanisms of modest efficiency are therefore biologically viable in such cases.

**Other Micromotors.** We briefly analyze the power conversion efficiency of two other micromotor systems. Self-thermophoretic motors are Janus microspheres half-coated with gold\(^\text{61,62}\) or Permalloy.\(^\text{55}\) They move autonomously in liquids due to a photothermally or magnetically induced asymmetric temperature gradient in the fluid surrounding the sphere. Jiang et al. found that 1 \(\mu\)m diameter silica microspheres half-coated with a 25 nm gold layer were propelled in water at 6 \(\mu\)m/s,\(^\text{58}\) yielding a mechanical power output of 3 \times 10^{-17} W (eq 6a).

The energy input was 1046 nm light, with 40 mW focused into a 9 \(\mu\)m diameter beam. Assuming an absorbance of ~1%\(^\text{57}\) for the gold cap, the power input was ~5 \times 10^{-8} W, yielding an energy efficiency on the order of 10^{-13}−10^{-14}.

Recently, Wang et al. have shown that metallic microrods undergo autonomous motion in ultrasonic standing waves at MHz frequency.\(^\text{65}\) These microrods move at axial speeds up to ~200 \(\mu\)m/s. The propulsion force is thought to arise from a small pressure difference induced by scattering of acoustic energy at the concave and convex ends of the asymmetric microrods. In these experiments, a piezoelectric transducer operating at 250 MW was coupled to a 25 cm² stainless steel plate. Because the microrods (3 \(\mu\)m long and 300 nm in diameter) are much smaller than the wavelength (~400 \(\mu\)m), the effective cross-sectional area for acoustic scattering can be roughly approximated as area as a circle whose diameter is the length of the rods (3 \(\mu\)m). Assuming that the acoustic power is distributed uniformly across the area of the cell, the power input to a single motor is ~7 \times 10^{-19} W. The mechanical power of the micromotors is estimated from the speed and drag force to be 2 \times 10^{-16} W (eq 6c). This gives an energy efficiency on the order of 10^{-7}.

### CONCLUSIONS

Both self-electrophoretic and bubble-powered catalytic micro-motors have efficiencies, defined as the fraction of chemical input energy that is converted to mechanical work, on the order of 10^{-9}−10^{-10}. These low efficiencies can be understood in semiquantitative terms as sequential stages of energy loss. A broader range of functionality could be available to catalytic micromotors if their efficiency could be increased by 3–4 orders of magnitude, including operation using dissolved O\(_2\) as the oxidant. The simplest path to increasing the efficiency of self-electrophoretic motors is to eliminate the background catalytic consumption of fuel, and this idea has already been demonstrated with bimetallic copper-halogen micromotors.\(^\text{43}\)

The low efficiency with which chemically generated potential gradients are converted to mechanical work (the third stage of energy loss, eq 17) suggests that self-electrophoretic micro-motors have an upper limit of efficiency of 10^{-3}−10^{-4}. Natural catalytic motors circumvent this problem by directly coupling...
the power stroke of chemical bond making or breaking to mechanical movement. It is interesting that both helical magnetic micromotors and acoustic motors are orders of magnitude more efficient than self-electrophoretic motors. They may have a broader range of potential applications, especially if some of the interesting functionality of catalytic motors (such as chemotaxis and other collective phenomena) can be realized with these propulsion mechanisms.

**ASSOCIATED CONTENT**

1. Supporting Information
A typical oxygen evolution measurement result, SEM images of the nanorods, numerical modeling details and results, and description of the calculation of motor speed based on fluid speed. This material is available free of charge via the Internet at http://pubs.acs.org.

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Notes
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